

Multi-brid DBI Inflation

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Abstract

We extended the multi-brid idea to the multi-field separable model with a non-canonical kinetic term, mainly the DBI model. Assuming a specific surface for the end of inflation and introducing new fields, enable us to find an explicit expression for the number of e-folds in terms of the new fields. By using δN formalism, we arrived at the cosmological parameters. We considered the DBI model for two different limits, viz., speed limit and constant sound speed.

I. INTRODUCTION

The inflation theory offers an intelligent approach to the basic problems of cosmology[1]. According to this theory, a rapid expansion era in the universe enlarges its physical length to the nearly 60 times the original size; this rapid expansion solves the old problems encountered in cosmology, such as flatness, horizon and monopole issues. The fluctuation of the inflaton, the field which drives inflation, induces the fluctuations in the energy density in such a manner that the curvature power spectrum is nearly scale invariant. There are several cosmological observations which support this theory. Recently, the Planck satellite observed the Cosmic Microwave Background(CMB) anisotropy and its polarization with a small angular resolution; it results concur with predictions of the inflation theory, in general[2–4].

Although the experimental tests support inflation; from the theoretical perspective, a proper understanding of the nature of the factors that produces the inflation is still unclear. In the simplest model, one field, the inflaton which is minimally coupled to gravity, rolls very slowly in a very flat potential. The flatness of the potential is essential to achieve enough inflation. One way to relax this condition is to change the dynamics of the inflaton. When the kinetic term in the Lagrangian of this field is non-canonic the slow-roll condition is not necessary anymore. For example when we have a Dirac-Born-Infeld (DBI) field, there is a fast-roll inflation[5]. Brane inflation is an example of this model in which the radial distance between a pair of D3- and anti D3-brane takes the role of the inflaton [6–19]. In general, it is possible to get inflation from a general class of non-standard kinetic terms, which is called k-inflation, in this model the inflation can exist even in the absence of the potential [20, 21]. However, the existence of more than one field can relax many limits on the single field models. In the multi-field models, co-operation between the fields can produce inflation, even if each field is not able to drive the inflation by itself, in such a way that the e-folding number and curvature perturbation are proportional to the number of fields. [22–27].

Regardless of the model of inflation, this era must end at some time t_f . For example, a waterfall mechanism can cease inflation on a specific surface in field space which, is called the end of inflation surface. By using the equation of end of inflation surface, the e-folding number can be expressed explicitly in terms of fields; this concept is known as multi-brid. [28, 29]. We generalize this idea to the separable multi-field models, assuming a general equation for this surface in terms of fields at t_f .

The rest of this paper is organized as follows: in Section(II), we present set-up for separable models. in the second section, we use $\delta\mathcal{N}$ formalism to arrive at observational parameter. In

Sections(III), we apply this method to the the multi-speed DBI in speed limit, and in Section(IV), it is applied to DBI with constant sound speed.

II. THE MODEL

We consider a separable action for N number of scalar fields with the non-canonical kinetic term; the action is given by

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_p^2 R + 2 \sum_I^N P_I(X_I, \phi_I)], \quad (1)$$

where X_I is the kinetic term of ϕ_I , $X_I = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi_I\nabla_\nu\phi_I$. Consider a spatially flat FRW background, $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$, the variation of action gives the field equations as follows,

$$\frac{d}{dt}(a^3 P_{I,X_I} \dot{\phi}_I) - a^3 P_{I,\phi_I} = 0. \quad (2)$$

As the fluctuation of each field is characterized by its own sound speed, $c_{S_I} = P_{I,X_I}/(P_{I,X_I} + 2X_I P_{I,X_I X_I})$, this is a "multi-speed" model [30, 31].

We are interested in extending the multi-brid [28, 29] idea to the model described above, in which the inflation ends by a waterfall mechanism on a specific surface, by defining new fields that move radially in the field space. The equations of motion are solvable, which enables the evolution of new fields in terms of the number of e-folds. Using $\delta\mathcal{N}$ formalism, the observational parameters such as spectral index and non-Gaussianity are produced.

Rewriting the equation of motion (2) as $\frac{1}{a^3 P_{I,\phi}} \frac{d}{dt} (a^3 P_{I,X} \dot{\phi}_I) = 1$, leads us to introduce new fields as follows:

$$\ln q_I(t) \equiv \int -\frac{\dot{a}}{a^4 p_{I,\phi_I}} \frac{d}{dt} (a^3 P_{I,X} \dot{\phi}_I) dt, \quad (3)$$

each q_I is a functional of t, through $a(t)$ given by the Friedmann equations and ϕ_I . In terms of these new fields, the equation of motion can be expressed very simply as,

$$\frac{d \ln q_I}{dt} = -H. \quad (4)$$

the right hand side is identical for all the fields; therefore, for each I and J we have,

$$\frac{\ln q_I}{\ln q_J} = \frac{\ln q_{I,f}}{\ln q_{J,f}}. \quad (5)$$

This indicates that in the space of q_I the motion is radial (see **Fig.1**).

For later convenience, the time variable is changed to the number of e-folds, \mathcal{N} , $d\mathcal{N} = -Hdt$. The minus sign shows that the number of e-folds is backward in time (at the end of inflation, it vanishes i.e. $\mathcal{N}_f = 0$). From (4), in terms of \mathcal{N} , the equation of motion for ϕ_I becomes,

$$\frac{d \ln q_I}{d\mathcal{N}} = 1, \quad (6)$$

which can be solved as

$$\mathcal{N} - \mathcal{N}_f = \ln q_I - \ln q_{I,f}. \quad (7)$$

It is worth mentioning that there is also another term contributing in \mathcal{N} , \mathcal{N}_c . This is derived from the fact that the end of the inflation surface is not a constant energy density surface[28], if we assume that the universe is radiation dominated. Immediately after the end of inflation, it is expressed by $\mathcal{N}_c = \frac{1}{4} \frac{\rho_f}{\rho_c}$, where $\rho = \sum_I X_I P_{IX}$ is the density, “f” and “c” refer to the end of the inflation surface and the surface of constant energy, respectively. Throughout the rest of the paper this term is ignored.

Suppose the end of inflation surface is known, for example, a waterfall mechanism terminates the inflation on a specific surface. This surface is determined by a relation between the fields in q space at $\mathcal{N}_f = 0$ as $f(\ln q_{I,f}) = \mathcal{M}^2$. By using this relation and the radial motion in q space, it is possible to arrive at the variation of the $q_{I,f}$ s in terms of the variation in q ’s. Throughout the rest of the paper we restrict ourselves to two fields which allow us to make some precise calculations. As the $\ln q_{I,f}$ s are not independent, at the end of the inflation there is only one degree of freedom in principle that can be characterized by only one parameter, which is denoted by θ . Therefore the $q_{I,f}$ are functions of θ , $\ln q_{I,f} = f_I(\theta)$, variation of (7) gives,

$$\delta \mathcal{N} = \delta \ln q_1 - \delta \ln q_{1,f} = \delta \ln q_2 - \delta \ln q_{2,f}. \quad (8)$$

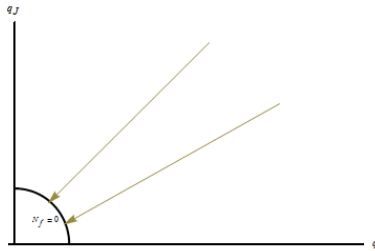


FIG. 1. In q 's space the motion is radial

Rewriting this equation as

$$\delta \ln q_1 - \delta \ln q_2 = \delta \ln q_{1,f} - \delta \ln q_{2,f}, \quad (9)$$

the equation given above can be expanded as follows:

$$\delta_1 \ln q_1 - \delta_1 \ln q_2 = \delta_1 \ln q_{1,f} - \delta_1 \ln q_{2,f} \quad (10)$$

$$\delta_2 \ln q_1 - \delta_2 \ln q_2 = \delta_2 \ln q_{1,f} - \delta_2 \ln q_{2,f},$$

where δ_1 and δ_2 refers to first- and second-order variations, respectively. Up to the second order, the variation of $\ln q_{I,f}$ can be obtained from the variations of $\ln q_{I,f} = f_I(\theta)$,

$$\delta \ln q_{I,f} = A_I(\delta_1 \theta + \delta_2 \theta) + A_{II} \frac{(\delta_1 \theta)^2}{2}, \quad (11)$$

with A_I and A_{II} being the first and second derivatives of f_I with respect to θ , equating the first-order and second-order terms of (10), respectively which gives the $\delta_1 \theta = (\delta \ln q_1 - \delta \ln q_2)/(A_1 - A_2)$ and $\delta_2 \theta = -(A_{11} - A_{22})/(A_1 - A_2)(\delta_1 \theta)^2/2$. Substituting (11) in (10) gives,

$$\delta_1 \ln q_{1,f} = \frac{A_1}{A_1 - A_2}(\delta_1 \ln q_1 - \delta_1 \ln q_2), \quad (12)$$

$$\delta_2 \ln q_{1,f} = \frac{1}{2} \frac{A_1 A_{22} - A_2 A_{11}}{(A_1 - A_2)^3}(\delta_1 \ln q_1 - \delta_1 \ln q_2)^2 + \frac{A_1}{A_1 - A_2}(\delta_2 \ln q_1 - \delta_2 \ln q_2), \quad (13)$$

where $\delta_2 \ln q_I = \frac{1}{2} \frac{\partial^2 \ln q_I}{\partial \phi_I^2} (\delta \phi_I)^2$. There is also a similar expression for q_2 .

A. $\delta \mathcal{N}$ formalism

Assume that an attractor solution exists and the $\delta \mathcal{N}$ formalism is applicable. The explicit expression of \mathcal{N} in terms of q_{IS} (7) allows us to compute $\delta \mathcal{N}$ in terms of field's variation. We assume slow variations at the horizon exit to ignore the $\dot{\phi}_I$. Substituting $\delta_1 \ln q_{I,f}$ in $\delta \mathcal{N}$ and ϕ_I 's variation, for $\ln q_{IS}$ variation as $\delta \ln q_I = (\ln q_I) \cdot \dot{\phi}$, gives

$$\delta_1 \mathcal{N} = H \left(\frac{A_2 \delta \phi_1 / \dot{\phi}_1 - A_1 \delta \phi_2 / \dot{\phi}_2}{A_1 - A_2} \right), \quad (14)$$

$$\delta_2 \mathcal{N} = \delta_2 \ln q_I - \delta_2 \ln q_{I,f}. \quad (15)$$

The second-order variation of e-folding number is as follows:

$$\begin{aligned} \delta_2 \mathcal{N} = & \frac{1}{2} H^2 \frac{A_2 A_{11} - A_1 A_{22}}{(A_1 - A_2)^3} \left(\frac{(\delta \phi_1)^2}{\dot{\phi}_1^2} + \frac{(\delta \phi_2)^2}{\dot{\phi}_2^2} - 2 \frac{\delta \phi_1}{\dot{\phi}_1} \frac{\delta \phi_2}{\dot{\phi}_2} \right) \\ & + \frac{1}{2} \left(\frac{-A_2}{A_1 - A_2} \frac{H \ddot{\phi}_1 - \dot{H} \dot{\phi}_1}{\dot{\phi}_1^3} (\delta \phi_1)^2 + \frac{A_1}{A_1 - A_2} \frac{H \ddot{\phi}_2 - \dot{H} \dot{\phi}_2}{\dot{\phi}_2^3} (\delta \phi_2)^2 \right), \end{aligned} \quad (16)$$

where we have replaced $\partial^2 q_I / \partial \phi_I^2$ with $(H \ddot{\phi}_I - \dot{H} \dot{\phi}_I) / \dot{\phi}_I^3$.

B. Cosmological Parameters

Assume that the two-point function of the scalar field fluctuations at the horizon exit is given by Gaussian distribution,

$$\langle \delta\phi_I \delta\phi_J \rangle_k = \left(\frac{H}{2\pi} \right)^2 |_{t_k} \delta_{IJ} \quad (17)$$

where t_k is the horizon crossing time of the co-moving wave-number k , such that $k\hat{c}_s = Ha$. \hat{c}_s is the maximum of sound speeds characterized by the final freezing scale. Using the $\delta\mathcal{N}$ formalism the curvature power spectrum and spectral index are given by

$$\mathcal{P}_S = \sum_I \mathcal{N}_{,I}^2 \left(\frac{H}{2\pi} \right)^2 |_{t_k} = \left(\left(\frac{H^4}{4\pi^2} \right) \frac{\frac{A_1^2}{\dot{\phi}_1^2} + \frac{A_2^2}{\dot{\phi}_2^2}}{(A_1 - A_2)^2} \right)_{t_k}, \quad (18)$$

$$\begin{aligned} n_s - 1 &= \frac{d \log \mathcal{P}_\zeta}{d \log k} = -2\epsilon_H + \frac{2}{H} \frac{\sum_{IJ} \dot{\phi}_I \mathcal{N}_{,I} \mathcal{N}_{,IJ}}{\sum_K \mathcal{N}_{,K}^2} \\ &= -2\epsilon_H - \frac{2}{H^2} \frac{H(A_2^2 \ddot{\phi}_1 / \dot{\phi}_1^3 + A_1^2 \ddot{\phi}_2 / \dot{\phi}_2^3) - \dot{H}(A_2^2 / \dot{\phi}_1^2 + A_1^2 / \dot{\phi}_2^2)}{A_1^2 / \dot{\phi}_2^2 + A_2^2 / \dot{\phi}_1^2}, \end{aligned} \quad (19)$$

where $\mathcal{N}_{,I} = \partial\mathcal{N}/\partial\phi_I$ and ϵ_H are defined as usual, as $\epsilon_H = -\frac{\dot{H}}{H^2}$. Another cosmological parameter is the local non-Gaussianity, given by

$$\begin{aligned} f_{NL}^{local} &= \frac{5}{6} \frac{\sum_{IJ} \mathcal{N}_{,I} \mathcal{N}_{,J} \mathcal{N}_{,IJ}}{\left(\sum \mathcal{N}_{,I}^2 \right)^2} \\ &= -\frac{5}{6} \frac{A_2 A_{11} - A_1 A_{22}}{A_1 - A_2} \frac{\left(A_1 / \dot{\phi}_1^2 + A_2 / \dot{\phi}_2^2 \right)^2}{\left(\left(A_1 / \dot{\phi}_1 \right)^2 + \left(A_2 / \dot{\phi}_2 \right)^2 \right)^2} \\ &\quad + \frac{5}{6} \frac{A_1 - A_2}{H^2} \frac{\left(H \left(A_1^3 \ddot{\phi}_2 / \dot{\phi}_2^5 - A_2^3 \ddot{\phi}_1 / \dot{\phi}_1^5 \right) - \dot{H} \left(A_1^3 / \dot{\phi}_2^4 - A_2^3 / \dot{\phi}_1^4 \right) \right)}{\left(\left(A_1 / \dot{\phi}_1 \right)^2 + \left(A_2 / \dot{\phi}_2 \right)^2 \right)^2}. \end{aligned} \quad (20)$$

Besides, there is an equilateral-type non-Gaussianity or a mixed shape because of the non-trivial kinetic term [30, 32–34], which we did not calculate explicitly here.

III. DBI IN SPEED LIMIT

Brane inflation, inspired by the string theory, is an example of models with a non-canonical kinetic term. In a simple model, the distance between a pair of D3 and anti-D3 brane, which are moving radially in a Calabi-Yau compactification, takes the role of an inflaton field. This model

includes a serious problem; it is unable to produce enough inflation due to the steepness of the potential between the brane and anti-brane. To resolve this problem, a warp factor is considered to flatten the potential, which comes from the movement of brane and anti-brane inside a warped throat. we considered two stacks of coinciding brane, inside a warped throat at different places, moving ultra relativistically towards the anti-branes located at the bottom of the throat. The distances between the two brane stacks and the anti-branes take on the role of the inflaton; therefore, this is a two-field model [35, 36]. We assume that the background metric is a flat FRW; the action, therefor is a follows:

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left(M_{pl}^2 R + 2 \sum_I [f_I^{-1} \left(1 - \sqrt{1 - f_I \dot{\phi}_I^2} \right) - V(\phi_I)] \right), \quad (21)$$

with $f_I = \frac{\lambda_I}{\phi_I^4}$. λ_I is proportional to the number of brane in each stack. We define *Lorentz factor* for each field as $\gamma_I = \frac{1}{\sqrt{1 - f_I \dot{\phi}_I^2}}$ which is the inverse of the sound speed for each field. The equation of motion is given by

$$\ddot{\phi}_I + 3H\gamma_I^{-2}\dot{\phi}_I + \frac{3}{2}\frac{f'_I}{f_I}\dot{\phi}_I^2 - \frac{f'_I}{f_I^2} + \gamma_I^{-3} \left(V'_I + \frac{f'_I}{f_I^2} \right) = 0 \quad (22)$$

In the speed limit $\gamma_I \gg 1$ and $\dot{\phi}_I^2 \approx f_I^{-1}$, this equation reduces to $\ddot{\phi}_I + \frac{3}{2}\frac{f'_I}{f_I}\dot{\phi}_I^2 - \frac{f'_I}{f_I^2} = 0$, which can be solved as $\phi_I(t) \approx \frac{\sqrt{\lambda_I}}{t}$, replacing in the Friedmann equations to get the Hubble constant as $H = \frac{\Lambda}{t}$, where $\Lambda^2 = \sum_I (\frac{\lambda_I m_I^2}{6M_P^2})$. Plugging this solution back into (22), up to the next leading correction, the solution is as follows:

$$\phi_I(t) \simeq \frac{\sqrt{\lambda_I}}{t} \left(1 - \frac{9H^2}{2m_I^4 t^2} \right), \quad (23)$$

once again replaced in the Friedmann equation gives the Hubble constant, $H = \frac{\Lambda}{t} \left(1 - \frac{(\Delta/\Lambda)^2}{t^2} \right)$, with $\Delta^2 = \frac{\sum_I \alpha_I m_I^2 \lambda_I}{6M_P^2}$. For the sake of simplicity, we introduce the small parameters, $\alpha_I = \frac{9H^2}{2m_I^4}$ and $\alpha = \frac{\Delta^2}{\Lambda^2}$. The e-folding number is obtained through the integration of H with respect to time,

$$\mathcal{N} = \Lambda \left(-\ln t - \frac{\alpha}{2t^2} \right) - \Lambda \left(-\ln t_f - \frac{\alpha}{2t_f^2} \right). \quad (24)$$

The equation given above enables us to designate the new fields as

$$\ln q_I = -\Lambda \ln t - \frac{\Lambda}{2} \frac{\alpha}{t^2}. \quad (25)$$

Assume that the surface of the end of inflation is, $g_1^2 \phi_{1,f}^2 + g_2^2 \phi_{2,f}^2 = \sigma^2$, with σ being a constant; up to the first order in the small parameters, we have,

$$\begin{aligned} A_1 &= -\Lambda \tan \theta \left(1 + \frac{\alpha \sigma^2}{g_1^2 \lambda_1} \cos^2 \theta \right), & A_2 &= +\Lambda \cot \theta \left(1 + \frac{\alpha \sigma^2}{g_2^2 \lambda_2} \sin^2 \theta \right), \\ A_{11} &= -\Lambda (1 + \tan^2 \theta) \left(1 - \frac{\alpha \sigma^2}{g_1^2 \lambda_1} \sin 2\theta \right), & A_{22} &= -\Lambda (1 + \cot^2 \theta) \left(1 + \frac{\alpha \sigma^2}{g_2^2 \lambda_2} \sin 2\theta \right). \end{aligned}$$

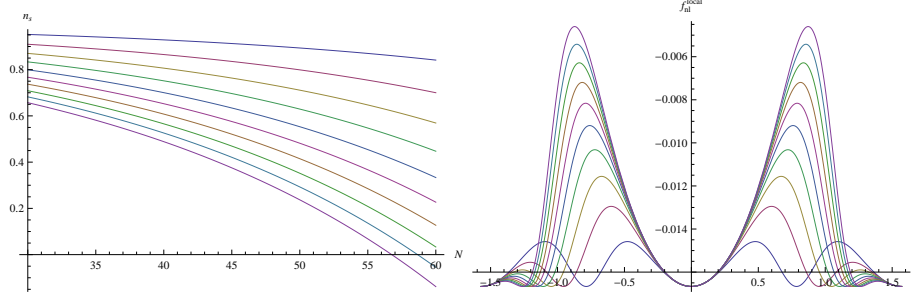


FIG. 2. We plot the spectral index on the left and the local non-Gaussianity on the right for the two-field DBI in the speed limit. We set the ratio of λ_1 to λ_2 from 1 to 10. θ is defined in the text for parameterizing the end of inflation surface; N is the number of e-folds.

With $\tan \theta = g_2 \sqrt{\lambda_2} / (g_1 \sqrt{\lambda_1})$. The power spectrum, spectral index, and local non-Gaussianities are as follows:

$$P_S = \frac{\Lambda^4}{4\pi^2 \lambda_1 \lambda_2} (\lambda_1 \sin^4 \theta + \lambda_2 \cos^4 \theta) |_{t_k} + \mathcal{O}(\alpha, \alpha_I), \quad (26)$$

$$n_S = 1 - \frac{4\alpha}{\Lambda t^2} m \quad (27)$$

$$f_{NL}^{local} = \frac{5}{3\Lambda} \frac{(-\lambda_1 \tan \theta + \lambda_2 \cot \theta)^2}{(\lambda_1 \tan^2 \theta + \lambda_2 \cot^2 \theta)^2} - \frac{5}{6\Lambda} (\tan \theta + \cot \theta) \frac{\lambda_1^2 \tan^3 \theta + \lambda_2^2 \cot^3 \theta}{(\lambda_1 \tan^2 \theta + \lambda_2 \cot^2 \theta)^2} + \mathcal{O}(\alpha, \alpha_I), \quad (28)$$

as before, t_k is the time of horizon crossing corresponding to the maximum of the sound speed. As the leading terms in spectral index cancel each other, we need to compute the first-order term. Up to zeroth-order of the small parameters, the P_S and f_{NL}^{local} have no H dependence. Assuming $m_1 \sim m_2 \sim 10^{-5} M_p$ and using $\Lambda \sim 50$ (which comes from the COBE normalization for the curvature power spectrum, $P_S \sim 2 \times 10^{-9}$) we get $\lambda_1 \sim \lambda_2 \sim 10^{14}$. We compare our model with that of the Planck data(2015)[3, 4], $f_{NL}^{local} = 0.8 \pm 5.0$ and $n_s = 0.968 \pm 0.006$, by appropriately choosing model parameters such as Λ , λ_I and \dots ; these parameters lie within the Planck range(Fig(2)).

IV. TWO FIELD DBI WITH CONSTANT SOUND SPEEDS

In [37], it is observed that for a single-field DBI model with $f(\phi) = f_0 \phi^{q+2}$, when the potential has a special form as $V(\phi) = V_0 \phi^{-q}$, there is an attractor solution with a constant sound speed, $c_s = \sqrt{3/(16f_0 V_0 + 3)}$. An extension of this solution with two fields is investigated in [38]. We extend this kind of solution to multi-speed case. Similar to the previous Section, we consider two stacks containing p_1 and p_2 branes moving towards $p_1 + p_2$ anti-branes at the bottom of the warped throat. We are interested in the potentials that include attractor the solutions with constant sound speed, $V_I = V_0 \phi_I^{-q}$. The form of the action is similar to (21) with $f_I(\phi) = f_0 \phi_I^{q+2}$,

where the equation of motion is rewritten as,

$$\ddot{\phi}_I + 3H\dot{\phi}_I - \frac{\dot{c}_{sI}}{c_{sI}} + c_{sI}V'_I - \frac{(1-c_{sI})^2}{2} \frac{f'_I}{f_I^2} = 0. \quad (29)$$

This equation has the solution as $c_{sI} = \sqrt{3/(q^2 f_0 V_0 + 3)}$ and $\phi_I = \phi_{I,0} t^{\frac{2}{4+q}}$ with $\phi_{0,I} = \left(\sqrt{\frac{1-c_{s,I}^2}{f_0}} \frac{2}{4+q} \right)^{\frac{2}{4+q}}$. Substituting in the Friedmann equations and keeping only the leading terms in late time, we arrive at

$$H = \tilde{\Lambda} t^{-\frac{q}{4+q}}, \quad (30)$$

where we define $\tilde{\Lambda}^2 = \frac{1}{3M_p^2} \left(\frac{4+q}{2} \right)^{\frac{2}{4+q}} \left(\frac{(\sum_I V_{0,I}(1-c_{s,I}^2))}{f_0} \right)^{\frac{1}{4+q}}$. Integration of H give us the number of the e-folding,

$$\mathcal{N} = -\tilde{\Lambda} \frac{4+q}{4} \left(t^{\frac{4}{4+q}} - t_f^{\frac{4}{4+q}} \right), \quad (31)$$

in which, as before, we select $\mathcal{N}_f = 0$. On comparing with (7), it directs us to define new fields as,

$$\ln Q_I = -\frac{\tilde{\Lambda}(4+q)}{4} t^{\frac{4}{4+q}}. \quad (32)$$

Assume that the inflation ends when $g_1^2 \phi_{1,f}^2 + g_2^2 \phi_{2,f}^2 = \sigma^2$, we have

$$\begin{aligned} A_1 &= -\frac{\beta_1 \sigma^2}{g_1^2} \sin 2\theta, & A_2 &= \frac{\beta_2 \sigma^2}{g_2^2} \sin 2\theta \\ A_{22} &= -2 \frac{\beta_1 \sigma^2}{g_1^2} \cos 2\theta, & A_{21} &= 2 \frac{\beta_1 \sigma^2}{g_1^2} \cos 2\theta, \end{aligned} \quad (33)$$

with $\beta_I = -\frac{4+q}{4} \tilde{\Lambda} \frac{1}{\phi_{0,I}^2}$ and $\tan \theta = \frac{g_2 \phi_{2,f}}{g_1 \phi_{1,f}}$. Replacing in (19),(20) and (21) gives,

$$P_S = -\frac{(4+q)\tilde{\Lambda}^3}{4\pi^2} \left(-\frac{4+q}{4} \right)^{\frac{q}{2}+1} \left(\frac{\tilde{\Lambda}}{\mathcal{N}} \right)^{\frac{q}{2}+1} \frac{\beta_1 \beta_2 (\beta_1 g_2^4 + \beta_2 g_1^2)}{(\beta_1 g_2^2 + \beta_2 g_1^2)^2}, \quad (34)$$

$$n_S = 1 - \frac{2-q}{2\mathcal{N}} \quad (35)$$

$$f_{nl}^{local} = -\frac{5}{12} \frac{(\beta_1 g_2^2 + \beta_2 g_1^2) (\beta_1 g_2^6 + \beta_2 g_1^6)}{\beta_2 g_1^4 + \beta_1 g_2^4} \frac{1}{\mathcal{N}} \quad (36)$$

Neither of the cosmological parameters depends on θ , which is characterized the surface of the end of inflation. As $ns = 0.968 \pm 0.006$, q must be in range $-2.56 \leq q \leq -1.12$ (Fig.3), we therefore select $q = -2.3$ which corresponds to $f = f_0 \phi^{-0.3}$ and $V = V_0 \phi^{2.3}$ and then plot the cosmological parameters (Fig.4).

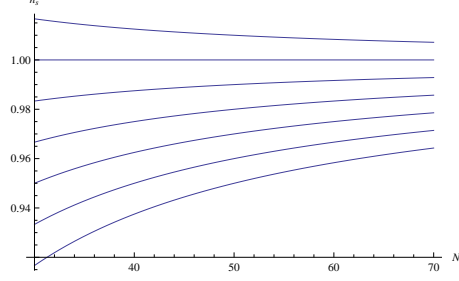


FIG. 3. We plot the spectral index for two-field DBI with constant sound speed, the parameter q goes from -3 to 3 . N_e is the number of e-folds.

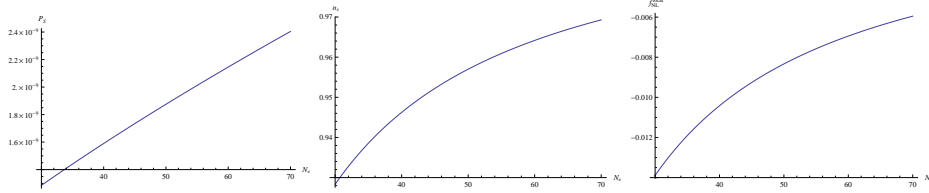


FIG. 4. We plot the power spectrum, spectral index and local non-Gaussianity for two-field DBI with constant sound speed and $q=-2.3$ versus the number of e-folds. Similar to the prior, the other parameters are selected as $v_1 = 5 \times 10^{-12}$, $v_2 = 10^{-12}$, $g_1 = g_2 = 3 \times 10^{-9}$, $c_1 = 0.1$ and $c_2 = 0.2$.

V. SUMMARY AND DISCUSSIONS

In this paper, we extended "multi-brid" inflation concept to a two-field separable model. We defined new fields that simplified the equation of motion, which enabled us to explicitly express the e-fold number in terms of this new field. We assumed that the end of inflation surface was known (a waterfall mechanism at the end of inflation can determine this surface). By equating the variations, order by order, we found the variation at the end of inflation surface in terms of the variations in the fields and replaced it in the e-fold variations. Using $\delta\mathcal{N}$ formalism produced the observational parameters such as scalar spectral index, its power spectrum and local non-Gaussianity. By using this set-up for the multi-speed DBI model in the speed limit and DBI with constant sound speed, we identified the cosmological parameters; on comparing with observation data, we obtained the numerical estimation for the parameters of these models. Such o DBI models are compatible with the Planck data.

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